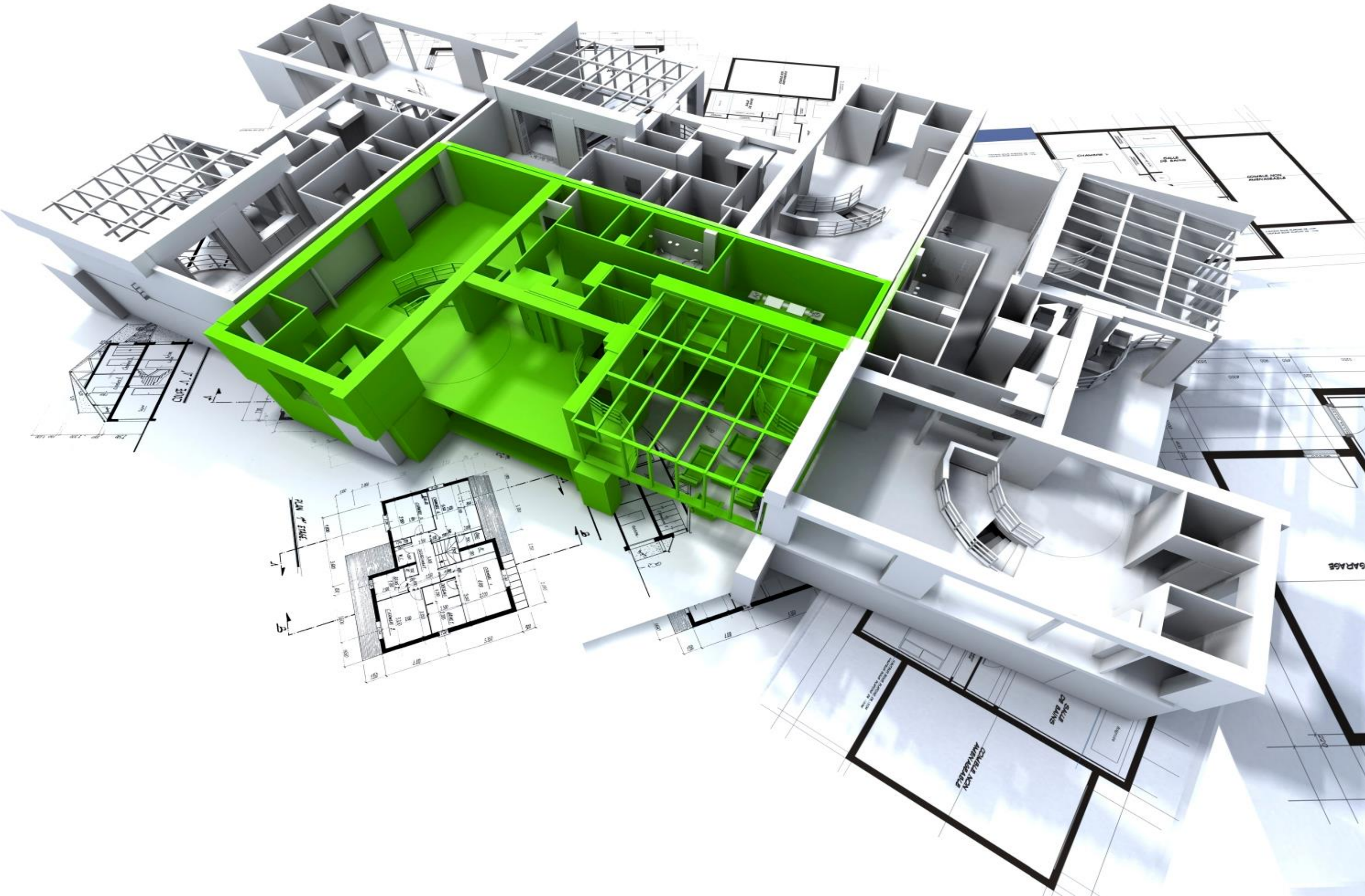


FLUIDS 3

AN INTRODUCTION TO COMPRESSIBLE FLOW

In this section we will some introductory topics in fast flows (called compressible flow). These include the regions of flow and isentropic flows.

FLUIDS 3

AN INTRODUCTION TO COMPRESSIBLE FLOW

OVERVIEW

In this section you'll learn about:

- Flow types and regimes (particularly with respect to compressible and fast flows).
- Isentropic flow
- The speed of sound
- Flow through nozzles
- A qualitative introduction to shockwaves

ASSUMED KNOWLEDGE FOR THIS SECTION

It is assumed that you already have a knowledge of the following topics:

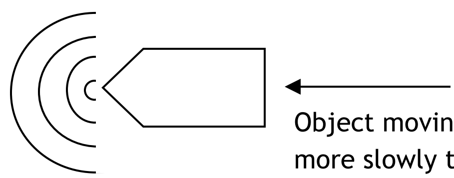
- *Basic fluid Mechanics – The Continuity Equation, Bernoulli's Equation and Forces in Fluids*
- *Fluid parameters – Density, Pressure and Viscosity.*
- *Fluid Statics – Hydrostaic pressure*

OBJECTIVE

To understand the ideas behind compressible flow and do basic isentropic calculations

TOPIC 1 – REVIEW OF FLOW TYPES

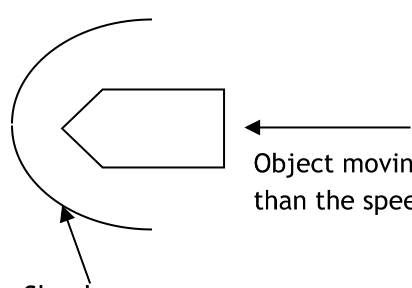
Liquids are not compressible. Gases can also be treated as incompressible, providing that they are travelling (or something is travelling through them) at less than 0.3 the speed of sound (known as Mach 0.3). Slow gases like these are incompressible because, when something moves through them, a pressure wave (a sound wave, if you like) travels ahead and "tells" the molecules to "get out of the way" – to rearrange themselves. Molecules have repulsive forces between them and don't "like" being pushed too close together - these forces are caused by the electrons in the fluid molecules repelling each other.



If, however, the object gets faster, eventually the molecules in front can't get out of the way in time (because the object is travelling so fast that there's little time for them to rearrange themselves) and they "bunch up." This region is called "compressible flow" and the density of the fluid is no longer a constant though it. Also, because the flow is now moving much faster, it heats up, and heat energy must be incorporated into our equations.

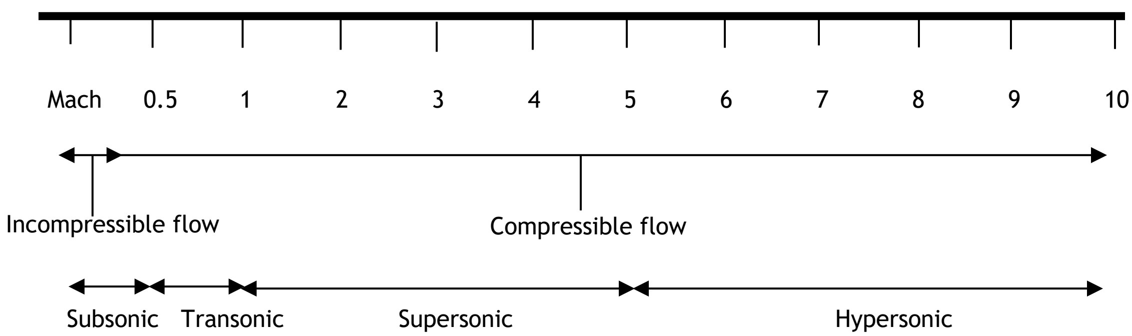
This situation gets progressively worse between Mach 0.3 and Mach 1. The result is that the answers given by our equations for incompressible flow (for example, Bernoulli's Equation) become less and less accurate.

When the flow or the moving object reaches the speed of sound, it is finally moving faster than the air molecules themselves. Because of this, they have no warning at all of its approach and the "bunching" becomes extreme – they form a dense, hot, viscous layer known as a shockwave. Shockwaves dominate supersonic flow and form when the flow encounters an obstacle or a supersonic object moves through them. As homework have a look at some videos on shockwave on youtube.



The final flow regime occurs when the flow gets so hot that the component gasses break-down and start reacting chemically with each other. This occurs above about Mach 5 in are and we say that the flow is Hypersonic.

Despite some difficulties in working out the formulae, the equations for compressible and supersonic flow are fairly simple. However, some complex problems do occur in the so called *Transonic region* - in which some of the flow is subsonic and some supersonic. This is because the flow is neither "one thing nor the other" – it displays traits of both types - and neither set of equations gives very accurate results (most of the data comes from experiment). Unfortunately, a lot of flow situations (such as modern passenger aircraft design) are in this region.



TOPIC 2 – ISENTROPIC FLOWS

Isentropic flows are simple compressible flows, without shockwaves. They occur mainly in two circumstances:

1. In the flow region between Mach 0.3 and Mach 1, before shockwaves form.
2. In supersonic flow, without shockwaves (there may be no shockwaves present, because the flow does not encounter an obstacle or because it changes its parameters very gently and smoothly).

Also, it is assumed that there is no energy added or leaking from the flow (so, no added heat, chemical reactions, etc). More formally, this means that such flows are:

- Adiabatic - There is no heat transfer between the fluid and its surroundings.
- Reversible - There are no frictional losses in the system (entropy is constant).

Although heat is not transferred, the temperature of the flow can of course vary - because of exchange between the kinetic energy and internal energy of the flow.

Let's now look at the important equations which govern isentropic flows:

a) The energy equation.

The energy equation is a statement of energy conservation like Bernoulli's equation. However, we need to add in an expression for heat energy:

$$E = CmT$$

And like the other energy equations in fluids, state it as an energy density (energy per cubic meter):

$$E_{density} = C\rho T$$

We can add this into Bernoulli's equation, remove the energy terms which are negligible in fast flows (the hydrostatic pressure terms) and then divide through by ρ to give:

$$C_p T + \frac{V^2}{2} = const$$

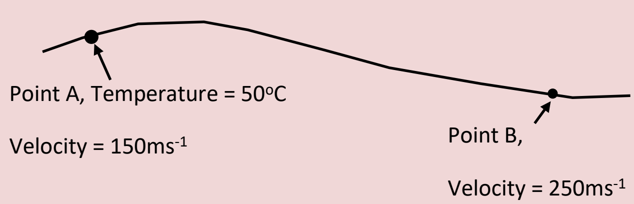
Along a streamline

Which is usually called the *Energy balance equation*.

This means that if the flow slows down, the kinetic energy decreases but the heat energy (and hence the temperature) of the flow increases. The opposite is also true of course. The meaning of C_p is explained in the section below.

TASK 1

Consider the two points on the streamline around a wing shown below:



Assuming the fluid is air (C_p = 1005 $JKg^{-1}K^{-1}$), what is the expected temperature (in ^0C) at point B, assuming isentropic flow?

b) Thermodynamic equations and relationships

To derive the remaining equations, we need some basic thermodynamics. One important and useful relationship is the equation of state:

$$p = \rho RT$$

The parameter R is the gas constant (287 J Kg K).

Another couple of simple formulae are useful.

If we have a fixed pressure of gas and we change its volume the internal energy changes as shown below.

$$C_p dT = -p dv \quad \text{Constant Pressure}$$

Alternatively, we could keep the volume fixed and change the pressure.

$$C_v dT = v dp \quad \text{Constant Volume}$$

Note that specific heats for each process are different. In fact, the ratio of these two specific heats occurs so often in the equations that it's given a special symbol.

$$\gamma = \frac{C_p}{C_v}$$

For air at standard conditions $\gamma \approx 1.4$

The relationship between C_p and C_v may also be expressed as

$$C_p T = C_v T + \frac{p}{\rho}$$

By simple manipulation be can also get:

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$C_v = \frac{R}{\gamma - 1}$$

c) Basic relationships between density, pressure and temperature

Let's start by dividing the two equations above.

$$\frac{-pdv}{vdp} = \frac{C_p}{C_v}$$

This can be rearranged.

$$\frac{dp}{p} = -\frac{C_v}{C_p}\frac{dv}{v}$$

or

$$\frac{dp}{p} = -\frac{1}{\gamma}\frac{dv}{v}$$

We can integrate this

$$\int_{p_1}^{p_2}\frac{dp}{p} = -\frac{1}{\gamma}\int_{v_1}^{v_2}\frac{dv}{v}$$

$$\Rightarrow \ln\frac{p_2}{p_1} = -\frac{1}{\gamma}\ln\frac{v_2}{v_1}$$

$$\Rightarrow \frac{p_2}{p_1} = \left(\frac{v_2}{v_1}\right)^{-\gamma}$$

Since we are talking about points in space (specific volumes) $v_1 = 1/\rho_1$ and $v_2 = 1/\rho_2$ so

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

c) Relationship between pressure and temperature

Using the equation of state $p = \rho RT$ rearranged to $\rho = p/RT$

$$\frac{p_2}{p_1} = \left(\frac{p_2}{RT_2}\frac{RT_1}{p_1}\right)^{\gamma}$$

Which reduces to

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

TASK 2

a) Assuming that the static pressure at point A in task 1 is 1 bar, calculate the pressure at point B. Calculate the density at points A and B (assume $\gamma \approx 1.4$).

b) The exit velocity from a rocket engine is given by:

$$v_e = \sqrt{2C_pT_c\left(1-\left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

Where the subscript c indicates conditions in the combustion chamber and e conditions at the exhaust. The process can be considered basically isentropic. Derive this equation (hint: you'll need the energy balance equation and the isentropic relationships from the last section).

TOPIC 3 – THE SPEED OF SOUND

Knowing the speed of sound in a fluid is important as it tells us when shockwaves will form. It's possible to derive an expression for the speed using only basic parameters. Let's start with the continuity equation.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Let us say that a sound wave is propagating up a streamline from position 1 to position 2. We'll say the areas are the same ($A_1 = A_2$) and that at position 2 the density is $\rho_1 + d\rho$ and the velocity is $V_1 + dV$. So (dropping the subscript)

$$\rho V = (\rho + d\rho)(V + dV)$$

We can multiply this out and arrange it, to produce

$$V = \rho\frac{dV}{d\rho}$$

Now, we can substitute Eulers equation ($dp = -\rho V dV$) into this

$$V = \frac{\rho}{d\rho}\frac{dp}{dV}$$

This can be shown to be equal to

$$\boxed{V = \sqrt{\frac{dp}{d\rho}}}$$

It's also fairly easy to show from this that

$$\boxed{V = \sqrt{\gamma RT}}$$

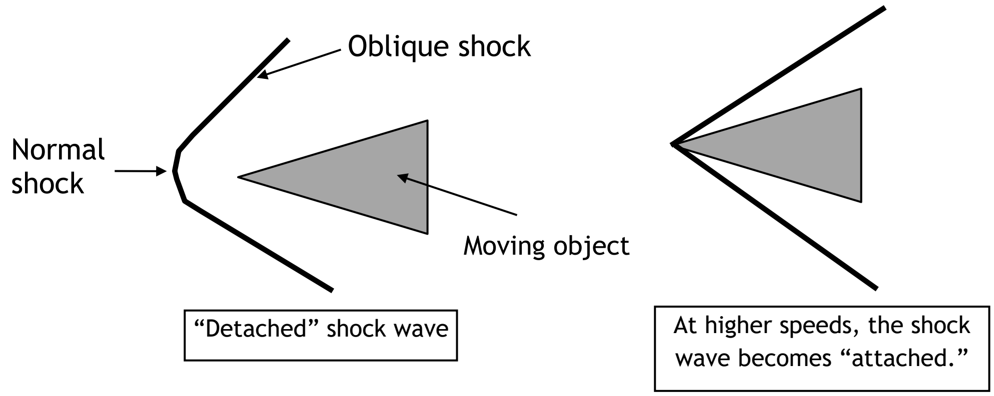
So the speed of sound in a fluid depends only on the temperature of the fluid. In books and other literature the speed is often given the symbol *a*. Another important symbol which relates to this is the Mach Number *M*. *M* is the speed relative to the speed of sound. So *M* = 0 means the fluid (or an object in the fluid) is not moving. *M* = 1 means that the object is moving at the speed of sound and *M* = 2 means it is moving at twice the speed of sound.

TASK 3

Calculate the speed of sound at points A and B in system described in tasks 1 and 2 and hence the Mach numbers of the flow at these points.

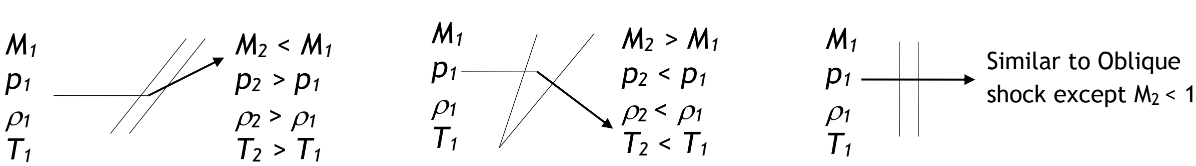
TOPIC 4 – SHOCKWAVES

When the speed of flow (or an object in the flow) exceeds the speed of sound for the fluid, shock waves are formed. These waves tend to come in three forms - “Normal” shocks which are at right angles to the flow, “Oblique” shocks which are at an angle to it. The third type - “Expansion fans” occur in the presence of more complex shapes. The first two types are shown in the examples below:



Expansion fans appear in more complex shapes when the body diverges away from the flow.

Flow across shock-waves is non-isentropic because the parameters change violently and the viscous properties of the fluid cause major thermal effects. The relationship between fluid parameters before and after the shocks are given by standard equations or by tables. The qualitative situation is shown below:



Oblique shock Expansion fan Normal shock

In the second year of the course you saw the effect of incompressible flow through a nozzle in the form of a Venturi. Let's now expand this to encompass compressible flow. We will start again with the continuity equation:

$$\rho AV = Constant$$

We can write this in terms of natural logs:

$$ln\rho + lnA + lnV = Constant$$

and differentiate it:

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

Now using Euler's equation ($dp = -\rho VdV$) rearranged for ρ:

$$-\frac{d\rho VdV}{dp} + \frac{dA}{A} + \frac{dV}{V} = 0$$

but $\frac{d\rho}{dp} = \frac{1}{a^2}$, so:

$$-\frac{VdV}{a^2} + \frac{dA}{A} + \frac{dV}{V} = 0$$

we can arrange this:

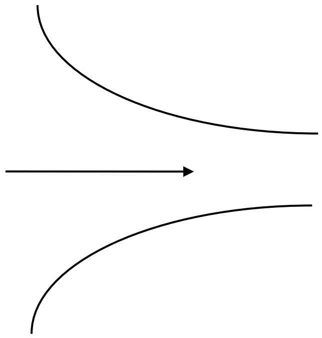
$$\frac{dA}{A} = \frac{VdV}{a^2} - \frac{dV}{V} = \left(\frac{V^2}{a^2} - 1\right)\frac{dV}{V}$$

or

$$\boxed{\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}}$$

This is known as the area-velocity relationship. Let us consider what it tells us about flow through nozzles.

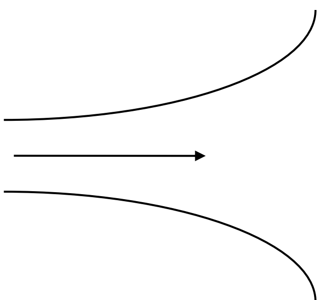
First a converging nozzle:



If $M < 1$ (which means that $(M^2 - 1) < 0$) and dA decreases, then dV must increase.

If $M > 1$ (which means that $(M^2 - 1) > 0$) and dA decreases, then dV must decrease.

and for a diverging nozzle

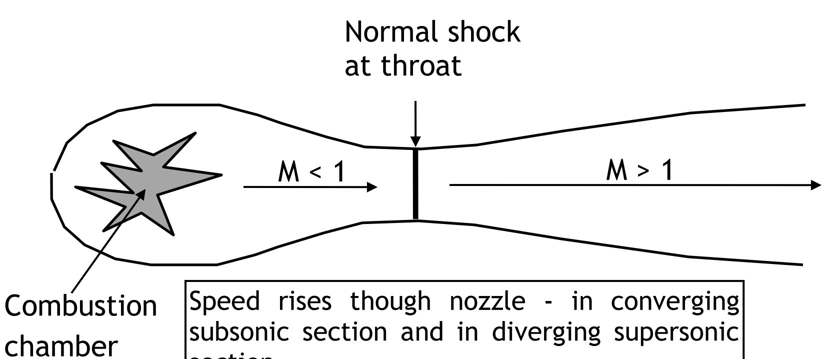


If $M < 1$ (which means that $(M^2 - 1) < 0$) and dA increases, then dV must decrease.

If $M > 1$ (which means that $(M^2 - 1) > 0$) and dA increases, then dV must increase.

So supersonic flow acts in exactly the opposite way to subsonic flow in nozzles.

A converging / diverging nozzle arranged so that $M = 1$ at the throat is known as a *de lavel* nozzle - this is the basis of (among other things) the rocket engine.



Speed rises though nozzle - in converging subsonic section and in diverging supersonic section

SUMMARY

- Compressible flow conditions apply when density in a flow can no longer be considered constant – typically in fast gaseous flows (or objects travelling fast through gases).
- At air speeds above about Mach 0.3 (110m/s in air at S.T.P.) the flow progressively becomes more compressible and incompressible equations like Bernoulli's become gradually less accurate until they no longer apply.
- Compressible flow can be divided up into several types:
  - Subsonic – with no shockwaves, below Mach 1
  - Standard supersonic – with shockwaves, above Mach 1
  - Transonic – which is a mixture of both types
  - Hypersonic – where the flow is so hot that chemical reactions occur, typically above Mach 5
- Isentropic flow is adiabatic and inviscid (reversible – no increase in entropy)
- Isentropic flows generally apply to compressible flows below Mach 1 and supersonic flow without shockwaves.
- Summary of isentropic equations:

Energy Balance equation	$C_p T + \frac{V^2}{2} = const$
Pressure, density and temperature relationships	$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}}$
Continuity equation	$\rho AV = const$
Equation of state	$p = \rho RT$

- Shockwaves dominate practical flow calculations above Mach 1
- Shockwaves are not isentropic and require their own equations to predict
- The three common phenomena associated with shockwaves are Normal shocks, Oblique Shocks and Expansion fans.
- Normal shocks occur when a flow drops from super to sub-sonic. Oblique shocks when an object causes the flow to turn into itself and Expansion fans when the flow turns away from itself.
- Flow travelling through a shockwave slows down, but most other parameters increase.
- Subsonic flow through a diverging nozzle speeds up, but it slows through a converging one (the opposite of subsonic flow) – this is the basis of the rocket engine (and a variety of other propulsion systems).